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An approach to calculate transient heat flow through multilayer spherical structures ☆

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Abstract

The transmission matrix and frequency characteristics of transient heat conduction through a multilayer spherical structure are presented by applying Laplace transforms in this paper. The frequency domain regression (FDR) [S.W. Wang, Y.M. Chen, Appl. Thermal Engrg. 21 (6) (2000) 683] method is introduced to construct some simple polynomial s-transfer functions from the frequency characteristics. The polynomial s-transfer functions are further used to calculate the transient heat flow of the spherical structure including its thermal response factors and Z-conduction transfer function (CTF) coefficients. This approach is very easy and simple to implement in programming and avoids the case of missing roots in finding the roots of characteristic equation. The examples and comparisons have demonstrated that this approach has high computation accuracy.

Keywords: Multilayer spherical structure; Frequency characteristics; Regression; Transient heat flow; Thermal response factors; CTF coefficients

1. Introduction

In order to evaluate building thermal performance and energy consumption, it becomes increasingly important to accurately predict transient heat gain through various building envelopes [1]. The thermal response factor method and the conduction transfer function (CTF) method are the most popular tools currently available for the hourly thermal load calculation through building envelopes in the design, simulation and energy analysis of building heating, ventilation and air-conditioning (HVAC) systems. The thermal response factors are the hourly series of heat flux at the inside and outside surfaces of a one-dimensional multilayer structure caused by unit triangular temperature pulses alternately applied to the inside and outside surface while holding the opposite surface at constant temperature [2]. The conduction transfer function is the Z-transform for the thermal response factors of the one-dimensional multilayer structure [3]. The major advantages of the two methods are that they are not numerical in the sense of finite difference techniques, and they do not require the periodic and linear boundary conditions for heat conduction calculation. They are also particularly well suited for use in computer programs together with the weather data recorded hourly.

In modern buildings, more and more semi-spherical and spherical envelopes such as the roofs of vaulted shape are constructed to implement some special functions or improve the artistic appearance of architectures. Some large semi-spherical structures are pieced up with many plane pieces of the same polygon, which can be calculated as plane constructions. Most structures of small size are pure semi-spherical or spherical. However, almost all relevant papers published are related to calculating the transient heat conduction through multilayer plane constructions [2–9]. Only Kusuda has discussed the calculation for the thermal response factors of a multilayer pure spherical structure by finding the roots of its characteristic equation [10]. The characteristic equation of a multilayer spherical structure is much more complicated than that of a multilayer plane wall. It is a tough task to calculate the thermal response factors and CTF coefficients of a multilayer plane wall by numerically finding the roots of its characteristic equation.

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Nomenclature

A, B, C, D	transmission matrix element	
a	thermal diffusivity	$m^2 \cdot s^{-1}$
b, d	CTF coefficients	
c_p	specific heat	$J \cdot kg^{-1} \cdot K^{-1}$
G	s- or z-transfer function	
h	heat transfer coefficient	$W \cdot m^{-2} \cdot K^{-1}$
K	thermal transmittance	$W \cdot m^{-2} \cdot K^{-1}$
l	term number of numerator	
M	transmission matrix	
m	term number of denominator	
N	number of frequency points	
n	layer number of a solid structure	
p	Laplace variable	
Q	heat flow	$W \cdot m^{-2}$
r	radius	m
s	Laplace variable or roots	
T	temperature	°C or K
t	time	s or h
X, Y, Z	thermal response factors	$W \cdot m^{-2} \cdot K^{-1}$
z	time delay operator	

Greek symbols

α, β	coefficient of polynomial s-transfer function	
λ	thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
$\Delta\tau$	time interval of discretization	s or h
δ	thickness	m
ρ	density	$kg \cdot m^{-3}$
σ	residue	$W \cdot m^{-1} \cdot K^{-1}$
ω	frequency	$rad \cdot s^{-1}$

Subscripts

i	integer count
in	inside
k	integer count
out	outside
X, Y, Z	outside, across and inside heat conduction

There is a risk to miss several roots in numerically searching, especially in case of two adjacent roots lying close together [2]. This may lead to incorrect results. It is a much tough and difficult work to calculate the transient heat flow through a multilayer spherical structure by searching for the roots of its characteristic equation. Although finite difference methods [8], state space methods [7,9] and time-domain methods [3] are presented for calculating the transient heat conduction through plane walls, they are quite complicated in programming and need considerable long computation time for this purpose. In the current building thermal load calculation, the hourly heat gain through the spherical structures is calculated approximately by being regarded as plane ones. For the spherical structure of small size, this approximation may result in a great error. Therefore, we need to develop an accurate and fast approach to calculate the transient heat flow through the spherical structures.

A simple, accurate and efficient frequency-domain regression (FDR) method is developed to calculate transient heat flow including thermal response factors and CTF coefficients of multilayer plane walls [11,12]. In this paper, the FDR method is introduced to calculate the thermal response factors and CTF coefficients of multilayer spherical structures. The FDR method is a numerical approach to implement easily and with very high accuracy. Through the case study of the FDR method, we will discuss the condition under which the multilayer pure semi-spherical and spherical structure can be approximately treated as a multilayer plane wall while the results holding the accuracy to meet the requirements of building thermal load calculation.

2. Transmission matrix of transient heat conduction through a spherical structure

2.1. Governing equations

Within the temperature varying range of building envelopes, we can assume that a single-layer spherical structure is homogeneous, isotropic and has constant thermal properties (i.e., λ, ρ and c_p), and that the transient heat conduction along its radial direction is one-dimensional. Thus, the heat conduction differential equation through the single-layer spherical structure is given in Eq. (1).

$$\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{2}{r}\frac{\partial T(r,t)}{\partial r} = \frac{1}{a}\frac{\partial T(r,t)}{\partial t}, \quad t > 0,\ r_1 < r < r_2 \tag{1}$$

where T is temperature, a and t are thermal diffusivity and time, respectively. r_1 and r_2 are the inside and outside radius of the spherical structure, respectively. The heat flow Q at an arbitrary time t and location r in the spherical structure is given in Eq. (2).

$$Q(r,t) = -\lambda \frac{\partial T(r,t)}{\partial r}, \quad t > 0,\ r_1 < r < r_2 \tag{2}$$

where λ is the thermal conductivity.

2.2. Transmission matrix of a single-layer spherical structure

With $\theta = rT$, Eq. (1) can be rewritten as follows.

$$\frac{\partial^2 \theta(r,t)}{\partial r^2} = \frac{1}{a}\frac{\partial \theta(r,t)}{\partial t} \tag{3}$$

Assuming that the temperature of the whole envelope holds at 0 °C when $t = 0$, i.e., $T(r,0) = 0$ (or $\theta(r,0) = 0$), Eqs. (4) and (5) can be obtained by applying Laplace transforms to the time variable t in Eqs. (3) and (2), respectively.

$$a\frac{\mathrm{d}^2\theta(r,s)}{\mathrm{d}r^2} - s\theta(r,s) = 0 \quad \text{and} \tag{4}$$

$$Q(r,s) = -\lambda\frac{\mathrm{d}T(r,s)}{\mathrm{d}r} \tag{5}$$

where $T(r,s)$, $\theta(r,s)$ and $Q(r,s)$ are the Laplace transforms of $T(r,t)$, $\theta(r,t)$ and $Q(r,t)$ with respect to the time variable t, respectively. Let $x = r - r_1$, Eq. (4) can be rewritten as Eq. (6).

$$a\frac{\mathrm{d}^2\theta(x+r_1,s)}{\mathrm{d}x^2} - s\theta(x+r_1,s) = 0 \tag{6}$$

By applying Laplace transforms to the variable x in Eq. (6), it can be expressed in Eq. (7).

$$a\big[p^2F(p,s) - p\theta(r_1,s) - \theta'(r_1,s)\big] - sF(p,s) = 0 \tag{7}$$

where $F(p,s)$ is the Laplace transform of $\theta(x+r_1,s)$ with respect to the variable x and $\theta'(r_1,s) = \frac{\mathrm{d}\theta(r,s)}{\mathrm{d}r}|_{r=r_1}$. Thus, $F(p,s)$ is found from Eq. (7) as follows.

$$F(p,s) = \frac{p^2}{p^2 - s/a}\theta(r_1,s) + \frac{1}{p^2 - s/a}\theta'(r_1,s) \tag{8}$$

The inverse Laplace transforms of $F(p,s)$ with respect to the Laplace variable p is given in Eq. (9).

$$\theta(x+r_1,s) = \mathrm{ch}\big(\sqrt{s/a}\,x\big)\theta(r_1,s) + \frac{1}{\sqrt{s/a}}\mathrm{sh}\big(\sqrt{s/a}\,x\big)\theta'(r_1,s) \tag{9}$$

We can obtain Eq. (10) by substituting $\theta(r,s) = rT(r,s)$ into Eq. (9),

$$\begin{aligned} rT(r,s) &= \mathrm{ch}\big(\sqrt{s/a}\,(r-r_1)\big)r_1T(r_1,s) \\ &\quad + \frac{1}{\sqrt{s/a}}\mathrm{sh}\big(\sqrt{s/a}\,(r-r_1)\big)\big[r_1T'(r_1,s) + T(r_1,s)\big] \end{aligned} \tag{10}$$

Thus, the Laplace transform of the temperature at the location r_2 with respect to the time variable t can be expressed in the Laplace transforms of the temperature and heat flow at the location r_1 as Eq. (11).

$$\begin{aligned} T(r_2,s) &= \frac{1}{r_2}\bigg[\mathrm{ch}\big(\sqrt{s/a}\,\delta\big)r_1 + \frac{1}{\sqrt{s/a}}\mathrm{sh}\big(\sqrt{s/a}\,\delta\big)\bigg]T(r_1,s) \\ &\quad - \frac{r_1}{\lambda r_2\sqrt{s/a}}\mathrm{sh}\big(\sqrt{s/a}\,\delta\big)Q(r_1,s) \end{aligned} \tag{11}$$

where $\delta = r_2 - r_1$, which is the thickness of the single-layer spherical structure. The Laplace transform of the heat flow at the location r_2 can be expressed in the Laplace transforms of the temperature and heat flow at the location r_1 as Eq. (12).

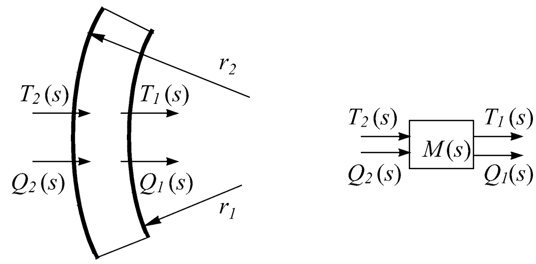

Fig. 1. The relationship between the temperature and heat flow at both sides of a single-layer spherical structure.

$$\begin{aligned} Q(r_2,s) &= -\lambda\bigg[\frac{\delta}{r_2^2}\mathrm{ch}\big(\sqrt{s/a}\,\delta\big) \\ &\quad + \frac{1}{r_2^2}\bigg(r_1r_2\sqrt{s/a} - \frac{1}{\sqrt{s/a}}\bigg)\mathrm{sh}\big(\sqrt{s/a}\,\delta\big)\bigg]T(r_1,s) \\ &\quad + \frac{r_1}{r_2^2}\bigg[r_2\mathrm{ch}\big(\sqrt{s/a}\,\delta\big) - \frac{1}{\sqrt{s/a}}\mathrm{sh}\big(\sqrt{s/a}\,\delta\big)\bigg]Q(r_1,s) \end{aligned} \tag{12}$$

From Eqs. (11) and (12), we can obtain the transmission equation of heat conduction through the single-layer spherical structure shown in Eq. (13) and Fig. 1, which relates the temperature and heat flow at both sides.

$$\begin{bmatrix} T(r_1,s) \\ Q(r_1,s) \end{bmatrix} = M(s)\begin{bmatrix} T(r_2,s) \\ Q(r_2,s) \end{bmatrix} = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix}\begin{bmatrix} T(r_2,s) \\ Q(r_2,s) \end{bmatrix} \tag{13}$$

The elements of the transmission matrix are given in Eq. (14),

$$\left.\begin{aligned} A(s) &= \frac{r_2}{r_1}\mathrm{ch}\big(\sqrt{s/a}\,\delta\big) - \frac{1}{r_1}\frac{1}{\sqrt{s/a}}\mathrm{sh}\big(\sqrt{s/a}\,\delta\big) \\ B(s) &= \frac{r_2}{r_1\lambda}\frac{1}{\sqrt{s/a}}\mathrm{sh}\big(\sqrt{s/a}\,\delta\big) \\ C(s) &= \frac{\lambda\delta}{r_1^2}\mathrm{ch}\big(\sqrt{s/a}\,\delta\big) \\ &\quad + \frac{\lambda}{r_1^2}\bigg(r_1r_2\sqrt{s/a} - \frac{1}{\sqrt{s/a}}\bigg)\mathrm{sh}\big(\sqrt{s/a}\,\delta\big) \\ D(s) &= \frac{r_2}{r_1}\mathrm{ch}\big(\sqrt{s/a}\,\delta\big) + \frac{r_2}{r_1^2}\frac{1}{\sqrt{s/a}}\mathrm{sh}\big(\sqrt{s/a}\,\delta\big) \end{aligned}\right\} \tag{14}$$

Equation $B(s) = 0$ is called the characteristic equation [2] of heat conduction through the single-layer spherical structure.

2.3. Transmission matrix of a multilayer spherical structure

Most building envelopes consist of more than three layers including the surface air films at both sides. The total transmission matrix of a multilayer spherical structure can be obtained by multiplying the transmission matrices of all layers including the surface air films at both sides. Assuming a spherical structure consists of n layers excluding its inside

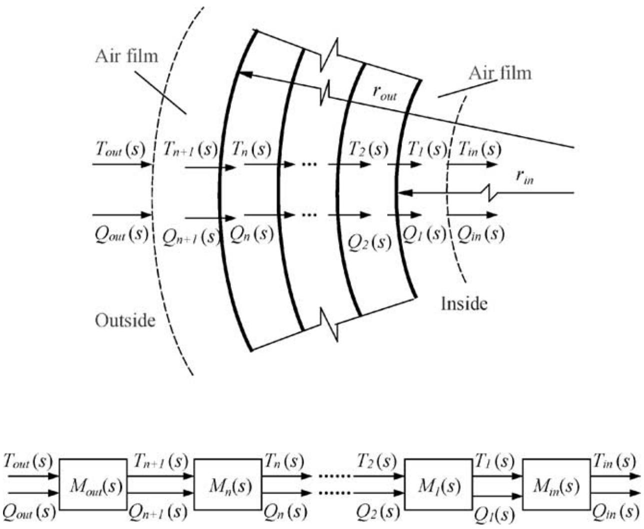

Fig. 2. The relationship between the temperature and heat flow of a multilayer spherical structure.

and outside surface air films, the relationship between the temperature and heat flow at both sides (shown in Fig. 2) can be expressed in Eq. (15).

$$\begin{bmatrix} T_{\text{in}}(s) \\ Q_{\text{in}}(s) \end{bmatrix} = \overline{M}(s) \begin{bmatrix} T_{\text{out}}(s) \\ Q_{\text{out}}(s) \end{bmatrix} = \begin{bmatrix} \bar{A}(s) & \overline{B}(s) \\ \overline{C}(s) & \overline{D}(s) \end{bmatrix} \begin{bmatrix} T_{\text{out}}(s) \\ Q_{\text{out}}(s) \end{bmatrix} \tag{15}$$

where $\overline{M}(s)$ is the total transmission matrix,

$$\overline{M}(s) = M_{\text{in}}(s) \cdot M_1(s) \cdots M_n(s) \cdot M_{\text{out}}(s) = \begin{bmatrix} A_{\text{in}}(s) & B_{\text{in}}(s) \\ C_{\text{in}}(s) & D_{\text{in}}(s) \end{bmatrix} \begin{bmatrix} A_1(s) & B_1(s) \\ C_1(s) & D_1(s) \end{bmatrix} \cdots \times \begin{bmatrix} A_n(s) & B_n(s) \\ C_n(s) & D_n(s) \end{bmatrix} \begin{bmatrix} A_{\text{out}}(s) & B_{\text{out}}(s) \\ C_{\text{out}}(s) & D_{\text{out}}(s) \end{bmatrix} \tag{16}$$

and $M_k(s)$ $(k = 1, 2, \ldots, n)$ is the transmission matrix of the kth layer, which elements are given in Eq. (14), subscripts in and out indicate the inside and outside of the structure, respectively. For the surface air films at both sides, the transmission matrix can be expressed in Eq. (17).

$$M_{\text{in}}(s) = \begin{bmatrix} 1 & -1/h_{\text{in}} \\ 0 & 1 \end{bmatrix}, \quad M_{\text{out}}(s) = \begin{bmatrix} 1 & -1/h_{\text{out}} \\ 0 & 1 \end{bmatrix} \tag{17}$$

where h is the heat transfer coefficient of the surface air film.

Applying matrix transform to Eq. (15), we can obtain the transmission equation (18) relating the temperatures to the heat flows at both sides.

$$\begin{bmatrix} Q_{\text{out}}(s) \\ Q_{\text{in}}(s) \end{bmatrix} = \begin{bmatrix} -G_X(s) & G_{Y'}(s) \\ -G_Y(s) & G_Z(s) \end{bmatrix} \begin{bmatrix} T_{\text{out}}(s) \\ T_{\text{in}}(s) \end{bmatrix} \tag{18}$$

where $G_X(s)$ and $G_Z(s)$ are the transfer functions of outside and inside heat conduction of the spherical structure, respectively, $G_Y(s)$ and $G_{Y'}(s)$ are the transfer functions of cross heat conduction from its outside to inside and from the inside to outside, respectively. For an n-layer spherical structure, $\bar{A}(s)\overline{D}(s) - \overline{C}(s)\overline{B}(s) = (r_{\text{out}}/r_{\text{in}})^2$, where r_{in} and r_{out} are its inside and outside radiuses, respectively. These transfer functions can be expressed in Eqs. (19)–(22).

$$G_X(s) = \bar{A}(s)/\overline{B}(s) \tag{19}$$

$$G_Y(s) = (r_{\text{out}}/r_{\text{in}})^2/\overline{B}(s) \tag{20}$$

$$G_{Y''}(s) = 1/\overline{B}(s) \tag{21}$$

$$G_Z(s) = \overline{D}(s)/\overline{B}(s) \tag{22}$$

Equation $\overline{B}(s) = 0$ is also called the characteristic equation [2,10] of heat conduction through the multilayer spherical structure. From Eqs. (14) and (16), it can be found that the characteristic equation $\overline{B}(s) = 0$ is much more complicated than that of a multilayer plane wall, which is presented in detail in Ref. [11]. For a multilayer plane wall, incorrect calculation might occur due to missing several roots in numerically searching for the roots of its characteristic equation, especially in case where two adjacent roots are close together [2]. For a multilayer spherical structure, it is much more difficult to perform transient heat conduction calculation than for a multilayer plane wall by finding the roots of the characteristic equation, and the incorrect calculation might take place in the same manner. Therefore, in this study, another simple approach implemented easily is introduced to calculate the transient heat conduction through a multilayer spherical structure from its frequency characteristics.

3. Frequency characteristics of heat conduction through a spherical structure

Substituting $j\omega$ $(j = \sqrt{-1})$ for s into Eqs. (19)–(22), we can obtained the complex functions $G_X(j\omega)$, $G_{Y'}(j\omega)$, $G_Y(j\omega)$ and $G_Z(j\omega)$, which are the frequency characteristics of outside, across and inside heat conduction, respectively [13]. They are all denoted as $G(j\omega)$. These frequency characteristics are complex functions and generally characterized by their amplitude $|G(j\omega)|$, which is the absolute value of $G(j\omega)$ and phase lag, $\arctan \frac{\text{imag}(G(j\omega))}{\text{real}(G(j\omega))}$, where $\text{real}(G(j\omega))$ and $\text{imag}(G(j\omega))$ are the real and imaginary components of $G(j\omega)$, respectively.

In practice, it is easy to correctly work out these four frequency characteristics in frequency domain without finding the embodied expressions of the four complex functions. The calculation procedure is that, first, the complex transmission matrices at N frequency points ($s_k = j\omega_k$, $k = 1, 2, \ldots, N$) are calculated by Eq. (14) for each layer of the multilayer spherical structure; second, the complex total transmission matrix at each frequency point is obtained by applying matrix multiplication as Eq. (16); finally, all four frequency characteristics with N frequency points are established through Eqs. (19)–(22).

4. Calculation for thermal response factors and CTF coefficients

Since it is much easier to correctly calculate the frequency characteristics of a multilayer spherical structure than to numerically search for the roots of its characteristic equation, we introduce the frequency domain regression (FDR) method [11] to construct some simple s-transfer functions from its frequency characteristics. In the FDR method, by minimizing the sum of the square error between the frequency characteristics of the spherical structure and the polynomial s-transfer function at all frequency points, the coefficients of the polynomial s-transfer function are easily obtained by solving a set of linear equations. The simple s-transfer functions are in the forms of the polynomial ratio of variable s. For short, they are called polynomial s-transfer functions. Through the polynomial s-transfer functions, it becomes much more easy, simple and correct to calculate the thermal response factors and CTF coefficients of a multilayer spherical structure. The calculation procedure is the same as that developed for a multilayer plane wall and is briefly reviewed here.

Given the properties λ, ρ, c_p and radius r of each layer in a multilayer spherical structure and the heat transfer coefficient h_{out} and h_{in} of its outside and inside surface air films, its four frequency characteristics with N frequency points can be easily calculated within the frequency range $[10^{-n_1}, 10^{-n_2}]$ needed to be concerned. For a building envelope, usually, $n_1 = 8$, $n_2 = 3$ and $N = 10(n_1 - n_2)$ [15]. The N frequency points are generated with equal logarithmic paces within the frequency range (i.e., $\omega_k = 10^{-n_1+(k-1)(n_1-n_2)/(N-1)}$ $(k = 1, 2, \ldots, N)$). The equivalent polynomial s-transfer function for each frequency characteristic, shown in Eq. (23), can be constructed by the FDR method, which is easily accomplished by solving a set of linear equations.

$$\widetilde{G}(s) = \frac{\beta_0 + \beta_1 s + \beta_2 s^2 + \cdots + \beta_r s^l}{1 + \alpha_1 s + \alpha_2 s^2 + \cdots + \alpha_m s^m} \tag{23}$$

where, α_i and β_i are real coefficients, l and m are the orders of the numerator and denominator, respectively. It is worth noting that the choosing method for the parameters of l, m and n_1, n_2 is based on the error analysis of frequency domain regression. By changing their values ($l = m = 4$–6, $n_2 = 3$–4, $n_1 = n_2 + 5$), we can find a minimal sum of square absolute error (SSAE) between the frequency characteristics of the spherical structure and the polynomial s-transfer function at all frequency points. The parameter values for the minimal SSAE are the optimal parameters we need. By a great number of case studies, it has been founded that the mean square absolute error for the optimal parameters is less than 10^{-14} [15].

In this study for the multilayer spherical structure in a building, the thermal response factors are the hourly-discretized series of the heat flux response of its polynomial s-transfer function to a unit triangular pulse excitation, which is a triangular temperature pulse of height $\phi = 1\,^\circ\mathrm{C}$ and base $2\Delta\tau$ at time $t = 0$ (conventionally, $\Delta\tau = 1$ hour). X, Y and Z are used to represent the outside, cross and inside thermal response factors of a spherical structure, respectively. Here, we take the cross thermal response factors Y as the example for their calculation. $Y(k)$ $(k = 0, 1, 2, 3, \ldots)$ are obtained by applying inverse Laplace transform to $\widetilde{G}_Y(s)/s^2$. The value of the factor $Y(0)$ at time $t = 0\Delta\tau$ is calculated using Eq. (24).

$$Y(0) = K + \sum_{i=1}^{m} \frac{\sigma_i}{\Delta\tau}\left(1 - e^{-s_i \Delta\tau}\right) \tag{24}$$

where K is the thermal transmission of the spherical structure. The subsequent factors $Y(k)$ at time $t = k\Delta\tau$ $(k = 1, 2, 3, \ldots)$ are calculated using Eq. (25).

$$Y(k) = -\sum_{i=1}^{m} \frac{\sigma_i}{\Delta\tau}\left(1 - e^{-s_i \Delta\tau}\right)^2 e^{-(k-1)s_i \Delta\tau} \tag{25}$$

where $-s_i$ is the ith root of the denominator of $\widetilde{G}_Y(s)$, and σ_i is the residue of $\widetilde{G}_Y(s)/s^2$ for the ith root. The thermal response factors $X(k)$ and $Z(k)$ $(k = 0, 1, 2, 3, \ldots)$ can be calculated using the same formulae as Eqs. (24) and (25) from the polynomial s-transfer functions $\widetilde{G}_X(s)$ and $\widetilde{G}_Z(s)$, which are constructed respectively from the transfer functions $G_X(s)$ and $G_Z(s)$ using the FDR method.

The conduction transfer function for the multilayer spherical structure is the Z-transform for the hourly-discretized series of the thermal response of its polynomial s-transfer function to the unit triangular pulse excitation, which can be expressed as Eq. (26).

$$G_Y(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_r z^{-l}}{1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_m z^{-m}} \tag{26}$$

where b_k and d_k are CTF coefficients. The formulae to calculate the CTF coefficients of a wall by employing its polynomial s-transfer functions have been addressed in detail in Refs. [14,15]. Therefore, it is very easy and simple to implement the calculation of the thermal response factors and CTF coefficients of multilayer spherical structures through the FDR method.

5. Validations and comparisons

In this study, various cases for multilayer spherical structures are calculated to validate the accuracy, simplicity and efficiency of the present approach by performing transient heat conduction calculation, including the thermal response factors and conduction transfer coefficients. Two examples are presented here to demonstrate its computation accuracy by comparisons with published results.

5.1. Thermal response factors

Kusuda [10] has provided three series of thermal response factors of a pure spherical structure by directly finding the

Table 1
Details of a double-layered brick spherical structure

	δ [mm]	λ [W·m^{-1}·K^{-1}]	a [m^2·s^{-1}]	r [mm]	R [m^2·kW^{-1}]
Inside air film	0			1524 (Inside radius)	0.1468
Common brick	101.5	0.727	4.9031E–7	1524	0.1396
Face brick	101.5	1.333	7.2256E–7	1625.5	0.0762
Outside air film	0			1727 (Outside radius)	0.0587

Table 2
Comparisons between thermal response factors

k	$X(k)^a$ [W·m^{-2}·K^{-1}]	$X(k)^b$ [W·m^{-2}·K^{-1}]	$Y(k)^a$ [W·m^{-2}·K^{-1}]	$Y(k)^b$ [W·m^{-2}·K^{-1}]	$Z(k)^a$ [W·m^{-2}·K^{-1}]	$Z(k)^b$ [W·m^{-2}·K^{-1}]
0	5.244716	5.244762	0.000771	0.000802	11.178113	11.178529
1	−0.914056	−0.914150	0.051959	0.051990	−3.008711	−3.009102
2	−0.428083	−0.428144	0.198693	0.198773	−1.377879	−1.378010
3	−0.274054	−0.274035	0.284558	0.284597	−0.928462	−0.928572
4	−0.195726	−0.195731	0.294361	0.294441	−0.693077	−0.693150
5	−0.149479	−0.149453	0.270634	0.270670	−0.544614	−0.544663
6	−0.118888	−0.118904	0.236717	0.236763	−0.440343	−0.440409
7	−0.096762	−0.096758	0.202314	0.202346	−0.361622	−0.361651
8	−0.079743	−0.079723	0.170972	0.170919	−0.299441	−0.299474
9	−0.066153	−0.066152	0.143674	0.143647	−0.249024	−0.249050
10	−0.055067	−0.055080	0.120392	0.120387	−0.207560	−0.207542
11	−0.045921	−0.045937	0.100736	0.100699	−0.173199	−0.173188
12	−0.038328	−0.038329	0.084227	0.084220	−0.144611	−0.144626
13	−0.032006	−0.032026	0.070397	0.070365	−0.120779	−0.120777
14	−0.026733	−0.026745	0.058826	0.058844	−0.100890	−0.100903

[a] Present approach;
[b] Kusuda.

roots of its characteristic equation. The spherical structure consists of an outside air film, a layer of face brick, a layer of common brick and an inside air film. In SI units, the thickness and thermal properties of all layers are listed in Table 1. Its polynomial s-transfer functions constructed by the FDR method are given as follows.

$$\begin{aligned}\widetilde{G}_X(s) = {} & \big[6.6201s^{-5} + 5.1634\text{e–}2s^{-4} + 9.5307\text{e–}5s^{-3} \\ & + 5.4995\text{e–}8s^{-2} + 8.4097\text{e–}12s^{-1} \\ & + 1.9627\text{e–}16\big] \\ & \times \big[s^{-5} + 8.2785\text{e–}3s^{-4} + 1.6450\text{e–}5s^{-3} \\ & \quad + 1.0580\text{e–}8s^{-2} + 2.0070\text{e–}12s^{-1} \\ & \quad + 7.5838\text{e–}17\big]^{-1}\end{aligned}$$

$$\begin{aligned}\widetilde{G}_Y(s) = {} & \big[-9.2140\text{e–}4s^{-5} + 5.2484\text{e–}6s^{-4} \\ & - 1.8011\text{e–}8s^{-3} + 4.1319\text{e–}11s^{-2} \\ & - 6.1162\text{e–}14s^{-1} + 4.5503\text{e–}17\big] \\ & \times \big[s^{-5} + 3.8205\text{e–}3s^{-4} + 5.1317\text{e–}6s^{-3} \\ & \quad + 2.7094\text{e–}9s^{-2} + 4.7507\text{e–}13s^{-1} \\ & \quad + 1.7582\text{e–}17\big]^{-1}\end{aligned}$$

$$\begin{aligned}\widetilde{G}_Z(s) = {} & \big[16.2350s^{-5} + 1.2302\text{e–}1s^{-4} + 2.2064\text{e–}4s^{-3} \\ & + 1.1976\text{e–}7s^{-2} + 1.6304\text{e–}11s^{-1} \\ & + 1.5834\text{e–}16\big] \\ & \times \big[s^{-5} + 8.3524\text{e–}3s^{-4} + 1.6859\text{e–}5s^{-3} \\ & \quad + 1.0923\text{e–}8s^{-2} + 2.0778\text{e–}12s^{-1} \\ & \quad + 7.8568\text{e–}17\big]^{-1}\end{aligned}$$

Its thermal response factors are further calculated by employing Eqs. (24), (25) and the above equations. The comparison is made between the values given by Kusuda and the approach developed in this study. The first 15 thermal response factors ($k = 0, 1, 2, \dots, 14$) are listed and compared in Table 2. The almost negligible difference between both series of the same thermal response factors has fully demonstrated the high accuracy of the approach developed in this study.

5.2. CTF coefficients

ASHRAE research project RP-472 provided a set of conduction transfer function (CTF) coefficients [5] corresponding to the representative constructions of 41 plane wall types and 42 plane roof types. These CTF coefficients in SI unit are given in the current *1997 ASHRAE Handbook of Fundamentals* [16]. In most current building thermal load calcu-

Table 3
Details of the wall group 6

	δ [mm]	λ [W·m^{-1}·K^{-1}]	ρ [kg·m^{-3}]	c_p [J·kg^{-1}·K^{-1}]	R[m^2·kW^{-1}]
Outside air film					0.0586
Stucco	25.39	0.6924	1858	8368	0.0367
High density concrete	101.59	1.7310	2243	8368	0.0587
Insulation	25.30	0.0433	32	8368	0.5846
Plaster	19.05	0.7270	1602	8368	0.0262
Inside air film					0.1206

Table 4
Comparisons between CTF coefficients

k		0	1	2	3	4
Plane wall	b_k	0.002870	0.053266	0.060031	0.007228	0.000050
	d_k	1.000000	−1.175800	0.300710	−0.015606	0.000006
$r_w/r_n = 1.005$	b_k	0.002882	0.053510	0.060328	0.007271	0.000050
	d_k	1.000000	−1.175411	0.300551	−0.015602	0.000005
$r_w/r_n = 1.01$	b_k	0.002896	0.053772	0.060620	0.0073058	0.000051
	d_k	1.000000	−1.175106	0.300482	−0.015595	0.000005
$r_w/r_n = 1.02$	b_k	0.002925	0.054294	0.061202	0.007375	0.000050
	d_k	1.000000	−1.174510	0.300361	−0.015587	0.000005
$r_w/r_n = 1.05$	b_k	0.003010	0.058854	0.062940	0.007583	0.000053
	d_k	1.000000	−1.172740	0.299989	−0.015560	0.000005
$r_w/r_n = 1.10$	b_k	0.003151	0.058433	0.065803	0.007923	0.000055
	d_k	1.000000	−1.169871	0.029936	−0.015507	0.000005
$r_w/r_n = 1.20$	b_k	0.003430	0.063514	0.071418	0.008586	0.000060
	d_k	1.000000	−1.164439	0.029809	−0.015384	0.000005

lations and building simulation programs, the transient heat gain through a pure semi-spherical or spherical structure is approximately evaluated by the CTF coefficients of the corresponding plane wall. In order to examine the rationality and condition of this approximation, a set of spherical structures with different radii ratios r_{out}/r_{in} are calculated. These spherical structures have the same conformation as the wall group 6 in *1997 ASHRAE Handbook of Fundamental*. That is, their thickness and thermal properties in each layer are the same as those of the wall group 6, listed in Table 3. The CTF coefficients of the wall group 6 are given by Harris and *ASHRAE Handbook of Fundamentals*. The radii ratios of the spherical structures are 1.005, 1.01, 1.02, 1.05, 1.10 and 1.20, respectively. The CTF coefficients of all spherical structures are obtained by the procedure presented in this study and compared with those of the plane wall, shown in Table 4. When the indoor temperature is kept at 24 °C and outdoor sol-air temperature is known, the hourly inside heat gain through the spherical structures is further calculated by the obtained CTF coefficients, shown in Table 5. The hourly heat gain given by *ASHRAE Handbook* for the plane wall under the same boundary conditions is also listed in Table 5. The results indicate that the hourly heat gain through the spherical structures is not in the proportion of r_{out}/r_{in} to that through the plane wall under the same boundary conditions. For the structures with the radii ratio r_{out}/r_{in} less than 1.05, the maximum relative errors between the heat gain through them and the plane wall are less than 5%, which is the permitted maximum error in building thermal load calculation. Therefore, for the case of this example, the spherical structure can be approximately regarded as a plane one only when its radius ratio is not greater than 1.05. Because of the its easiness, simplicity and high accuracy, the transient heat flow through all pure spherical structures can be calculated directly and accurately by the procedure presented in this study, and does not need to be approximately treated as a plane one.

6. Conclusions

Through the total transmission matrix of transient heat conduction through a multilayer pure spherical structure, it can be concluded that its characteristic equation is a very complicated one containing many hyperbolic functions. It is not only quite difficult to implement the calculation of the heat conduction by finding the roots of its characteristic equation, but also incorrect calculation might take place due to missing several roots in the same manner as a multilayer plane wall. It is easy and simple to calculate its frequency characteristics from the transmission matrix of a multilayer pure spherical structure. Some polynomial s-

Table 5
Comparisons between hourly inside heat gain ($W \cdot m^{-2}$)

Time (h)	Sol-air temperature (°C)	Plane wall	Spherical structure, r_{out}/r_{in}					
			1.005	1.01	1.02	1.05	1.10	1.20
0	25.0	13.403	13.436	13.468	13.532	13.721	14.026	14.601
1	24.4	11.646	11.671	11.695	11.742	11.883	12.108	12.528
2	24.4	9.999	10.016	10.033	10.067	10.165	10.322	10.611
3	23.8	8.517	8.528	8.539	8.561	8.625	8.725	8.908
4	23.3	7.203	7.21	7.216	7.229	7.265	7.321	7.419
5	23.3	6.016	6.019	6.021	6.026	6.04	6.06	6.09
6	23.8	4.960	4.959	4.958	4.957	4.953	4.943	4.919
7	25.5	4.081	4.079	4.076	4.07	4.053	4.023	3.964
8	27.2	3.473	3.47	3.466	3.459	3.437	3.401	3.329
9	29.4	3.209	3.206	3.203	3.197	3.18	3.151	3.097
10	31.6	3.304	3.304	3.303	3.302	3.298	3.293	3.286
11	33.8	3.755	3.758	3.761	3.767	3.786	3.817	3.884
12	36.1	4.523	4.531	4.539	4.555	4.603	4.683	4.844
13	43.3	5.584	5.598	5.612	5.64	5.722	5.86	6.133
14	49.4	7.162	7.184	7.206	7.25	7.38	7.595	8.019
15	53.8	9.498	9.531	9.563	9.629	9.825	10.148	10.782
16	55.0	12.441	12.488	12.534	12.626	12.901	13.353	14.239
17	52.7	15.591	15.651	15.71	15.829	16.184	16.766	17.904
18	45.5	18.406	18.477	18.548	18.689	19.108	19.797	21.138
19	30.5	20.260	20.337	20.413	20.565	21.018	21.76	23.201
20	29.4	20.392	20.466	20.539	20.685	21.12	21.831	23.206
21	28.3	19.019	19.083	19.146	19.273	19.649	20.261	21.439
22	27.2	17.157	17.21	17.262	17.366	17.674	18.174	19.131
23	26.1	15.247	15.289	15.330	15.413	15.658	16.054	16.807
Maximal relative error (%)			0.38	0.76	1.51	3.74	7.41	14.52

transfer functions can also be easily constructed from the frequency characteristics by utilizing the FDR method. This makes it quite easy to accurately calculate the transient heat flow including thermal response factors and CTF coefficient through the polynomial s-transfer functions. The examples have demonstrated that the approach developed in this study has very high accuracy. Therefore, in building thermal performance evaluation and building simulation programs, the transient heat flow through a multilayer pure spherical structure can be calculated easily, efficiently and accurately by this approach. It does not need to approximately treat a pure semi-spherical or spherical structure as a plane one.

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